universe+

UNIVERSE+ Online Seminar

Agnese Bissi
"AdS Amplitudes from CFT"

Part 1

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AdS Loops Amplitudes From CFT

Agnese Bissi
(ICTP & Uppsala University)

Based on:

Work in progress w/ G. Fardelli & M.R. Khansari

& also different collaborations w/ G. Fardelli, A. Georgoudis, A. Manenti

IN THE LAST YEARS THERE HAS BEEN AN INCREASING INTEREST IN UNDERSTANDING HOW TO CLASSIFY AND DEFINE CONFORMAL FIELD THEORIES IN $d \geq 3$.

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ONE IDEA: CONFORMAL BOOTSTRAP

CLASSIFY CFTs USING SYMMETRIES AND CONSISTENCY CONDITIONS

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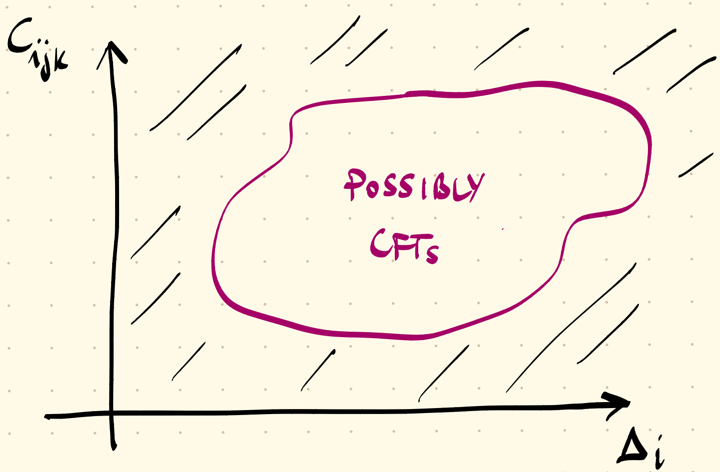
SPECIFIED BY $\{\Delta_i, C_{ijk}\}$

C_{ijk} → OPE COEFFICIENTS

Δ_i → CONFORMAL DIMENSIONS



// → NO CONSISTENT CFTs

THIS APPROACH IS PARTICULARLY INTERESTING AND SUCCESSFULL

IN THE CONTEXT OF THE AdS/CFT CORRESPONDENCE.

HOW TO IDENTIFY
HOLOGRAPHIC CFTs
USING BOOTSTRAP TECHNIQUES

CONNECTIONS
WITH AdS
SIDE & CONSEQUENCES
FOR QUANTUM GRAVITY

AdS/CFT CORRESPONDENCE

AdS SCATTERING AMPLITUDES $\longleftrightarrow$ CFT CORRELATORS

LOOP EXPANSION (IN G_N) $\longleftrightarrow$ $\frac{1}{N}$ EXPANSION

$\downarrow$

c^{-1} EXPANSION

CENTRAL CHARGE OF THE CFT

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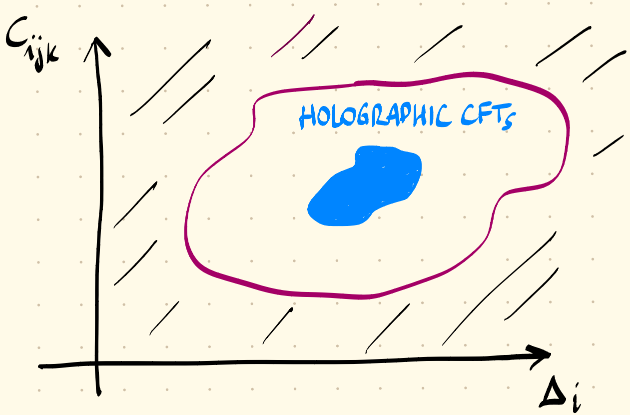


CONFORMAL FIELD THEORIES

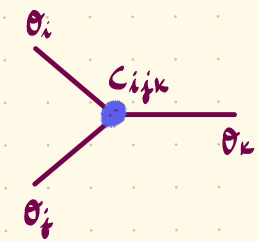
- LOCAL OPERATORS $\quad \mathcal{O}_{\Delta,\ell}$ ← SPIN

  ↑ DIMENSION

- THREE POINT FUNCTIONS $\quad C_{ijk}$

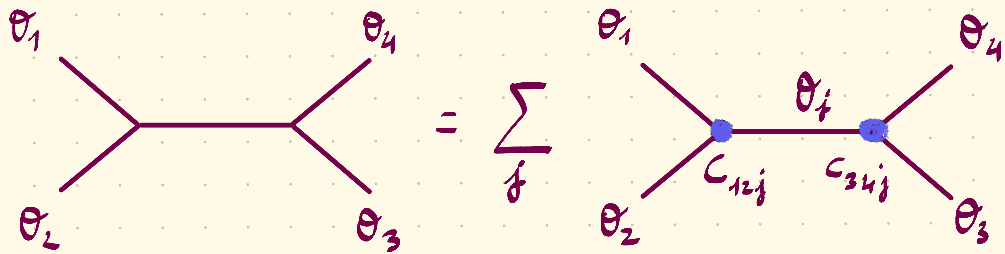
  OPE COEFFICIENTS

  ↳ $\mathcal{O}_i \times \mathcal{O}_j \sim \boxed{C_{ijk}}\, \mathcal{O}_k + \dots$



WITH THIS PIECE OF INFORMATION, IT IS POSSIBLE TO CONSTRUCT ALL HIGHER POINT FUNCTIONS.

- FOR INSTANCE 4 POINT FUNCTIONS

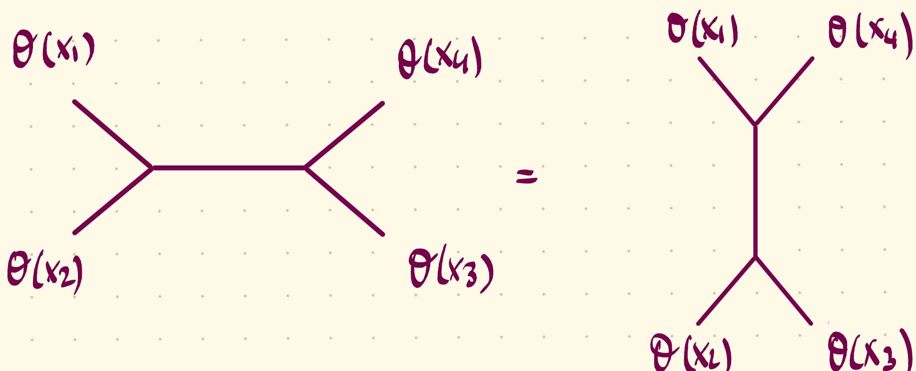


- THE SAME FOR HIGHER POINT FUNCTIONS

Conformal Bootstrap Philosophy:

- Use unitarity of the theory $\quad \Delta \geqslant \Delta^u(d,\ell)$
  $c^2 \geqslant 0$

- Use associativity of the operator product expansion



- If there are extra symmetries, use also then (SUSY, $O(N)$ ...)

AdS	CFT
FIELDS ϕ_i	SINGLE TRACE OPERATORS $\mathcal{O}_i$
GRAVITON $g_{\mu\nu}$	STRESS TENSOR $T_{\mu\nu}$
MULTI PARTICLE STATES (COMPOSITES)	HIGHER TRACES $[\mathcal{O}_i, \mathcal{O}_j \dots \mathcal{O}_k]$
MASS $m_i^2 = \Delta_i(\Delta_i - d)$	CONFORMAL DIMENSION Δ_i

picture

AIM OF THIS TALK:

- UNDERSTAND HOW TO USE THE CFT DESCRIPTION TO CONSTRAIN / CONSTRUCT THE AdS AMPLITUDES
- FOCUS ON LARGE N LIMIT

$\updownarrow$

SMALL G_N EXPANSION

Large N expansion

$$\langle \mathcal{O}(x_1)\,\mathcal{O}(x_2)\,\mathcal{O}(x_3)\,\mathcal{O}(x_4)\rangle = \frac{\mathcal{G}(u,v)}{x_{12}^{2\Delta_\mathcal{O}}\, x_{34}^{2\Delta_\mathcal{O}}}$$

Large N expansion

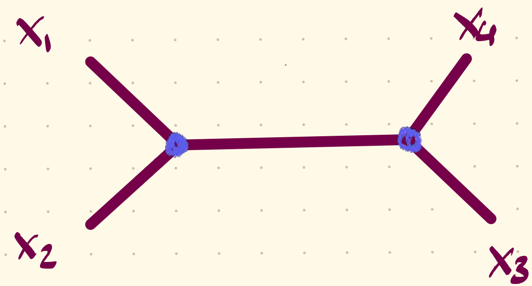
$$\langle \mathcal{O}(x_1)\,\mathcal{O}(x_2)\,\mathcal{O}(x_3)\,\mathcal{O}(x_4)\rangle = \frac{\mathcal{G}(u,v)}{x_{12}^{2\Delta_\mathcal{O}}\, x_{34}^{2\Delta_\mathcal{O}}}$$

$$\mathcal{G}(u,v) = \sum_{\Delta,\ell} a_{\Delta,\ell}\, \underbrace{u^{\frac{\Delta-\ell}{2}}\, g_{\Delta,\ell}(u,v)}_{\text{CONFORMAL BLOCKS}} \qquad d=4$$

$$\mathcal{G}(u,v) = \sum_{\Delta,\ell}$$

Large N expansion

$$\langle \mathcal{O}(x_1)\,\mathcal{O}(x_2)\,\mathcal{O}(x_3)\,\mathcal{O}(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2\Delta_\mathcal{O}}\, x_{34}^{2\Delta_\mathcal{O}}}$$



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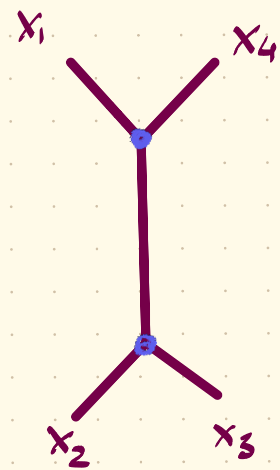


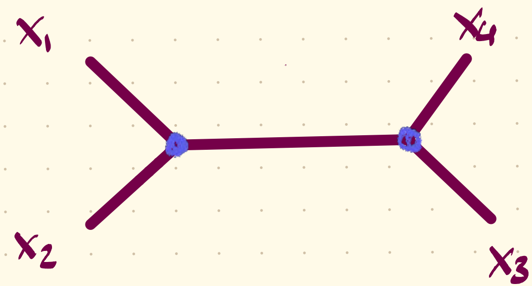
$$u = \frac{x_{12}^2\, x_{34}^2}{x_{13}^2\, x_{24}^2}$$

$$v = \frac{x_{14}^2\, x_{23}^2}{x_{13}^2\, x_{24}^2}$$

$$\frac{g(u,v)}{x_{12}^{2\Delta_\mathcal{O}}\, x_{34}^{2\Delta_\mathcal{O}}} = \frac{g(v,u)}{x_{14}^{2\Delta_\mathcal{O}}\, x_{34}^{2\Delta}}$$

Large N expansion

$$\langle \mathcal{O}(x_1)\,\mathcal{O}(x_2)\,\mathcal{O}(x_3)\,\mathcal{O}(x_4)\rangle = \frac{\mathcal{G}(u,v)}{x_{12}^{2\Delta_{\mathcal{O}}}\, x_{34}^{2\Delta_{\mathcal{O}}}}$$



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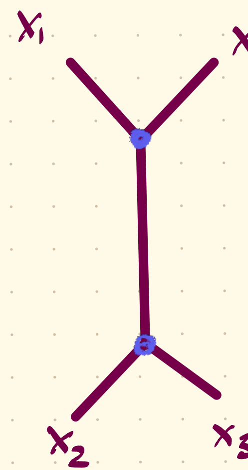


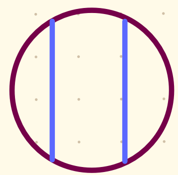
$$u = \frac{x_{12}^2\, x_{34}^2}{x_{13}^2\, x_{24}^2}$$

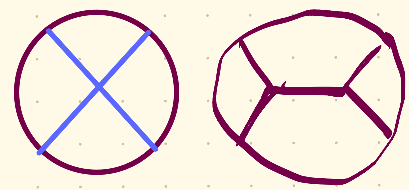
$$v = \frac{x_{14}^2\, x_{23}^2}{x_{13}^2\, x_{24}^2}$$

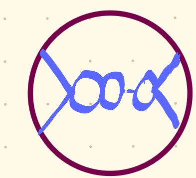
$$\mathcal{G}(u,v) = \mathcal{G}(v,u)\left(\frac{u}{v}\right)^{\Delta_{\mathcal{O}}}$$

- EXPAND THE CORRELATOR AND OPE DATA IN THE LARGE N LIMIT

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\,\mathcal{G}^{(1)}(u,v) + \dots + \frac{1}{N^{2k}}\,\mathcal{G}^{(k)}(u,v)$$







$$\Delta = \Delta^{(0)} + \frac{1}{N^2}\,\gamma^{(1)} + \dots + \frac{1}{N^{2k}}\,\gamma^{(k)}$$

$$a_{\Delta,\ell} = a^{(0)} + \frac{1}{N^2}\,a^{(1)} + \dots + \frac{1}{N^{2k}}\,a^{(k)}$$

⋮ higher trace + new operators

- DEPENDING AT WHICH ORDER WE ARE, THE INTERMEDIATE OPERATORS ARE DIFFERENT

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- GENERICALLY:

→ AT LEADING ORDER N^0 ONLY DOUBLE TRACE OPERATORS → $[\theta\theta]_{n,\ell} \equiv \theta \, \Box^n \partial_{\mu_1 \dots \mu_\ell} \theta$

$\Delta^{(0)} = 2\Delta_\theta + 2n + \ell$

- DEPENDING AT WHICH ORDER WE ARE, THE INTERMEDIATE OPERATORS ARE DIFFERENT

$$\boxed{\mathcal{G}(u,v) = \sum_{\Delta,\ell} a_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}} g_{\Delta\ell}(u,v)}$$

- GENERICALLY:

→ AT LEADING ORDER N^0 ONLY DOUBLE TRACE OPERATORS → $[\mathcal{O}\mathcal{O}]_{n,\ell} \equiv \mathcal{O}\,\Box^n \partial_{\mu_1\ldots\mu_\ell}\mathcal{O}$

$\Delta^{(0)} = 2\Delta_{\mathcal{O}} + 2n + \ell$

→ AT ORDER N^{-2}

- CORRECTION TO DOUBLE TRACES → $\gamma^{(1)}_{n,\ell}$, $a^{(1)}_{n,\ell}$
- NEW OPERATORS → $\tilde{\mathcal{O}}$ $\quad \langle \mathcal{O}\mathcal{O}\tilde{\mathcal{O}}\rangle \propto \frac{1}{N}$

→ AT HIGHER ORDERS K

- K-th CORRECTION TO DOUBLE TRACES → $\gamma^{(K)}_{n,\ell}$ $a^{(K)}_{n,\ell}$
- HIGHER TRACES $[\phi\phi\ldots\phi]$
-
-

- INSERT THESE EXPANSIONS IN THE CONFORMAL BLOCK DECOMPOSITION:

$$\Delta = 2\Delta_\phi + 2n + \ell$$

$$\mathcal{G}^{(0)}(u,v) = \sum_{n,\ell} a^{(0)}_{n,\ell}\, u^{\Delta_\phi+n}\, g_{n,\ell}(u,v)$$

$$\mathcal{G}^{(1)}(u,v) = \sum_{n,\ell} u^{\Delta_\phi+n}\left(a^{(1)}_{n,\ell} + \frac{1}{2} a^{(0)}_{n,\ell}\, \gamma^{(1)}_{n,\ell}\left(\log u + \frac{\partial}{\partial n}\right) g_{n,\ell}(u,v)\right)$$

$$\vdots$$

$$\boxed{\mathcal{G}^{(k)}(u,v) \supset \sum_{n,\ell} \frac{1}{2^k k!} u^{\Delta_0+n} a^{(0)}_{n,\ell} \left(\gamma^{(1)}_{n,\ell}\right)^k \log^k u \; g_{n,\ell}(u,v)}$$

THIS TALK: UNDERSTAND HOW MUCH INFORMATION (BOTH OF THE OPE DATA AND OF THE AMPLITUDE) WE CAN LEARN FROM THE KNOWLEDGE OF THIS TERM.

WE WILL CONSIDER ONLY ϕ^4 TYPE INTERACTIONS

OTHER MOTIVATIONS:

- TOY MODEL TO UNDERSTAND THE BEHAVIOUR OF HIGHER TRACE OPERATORS OF THE FORM $[\theta\theta\theta]$ (DIFFERENT FROM KRAVCHUK & MANN)

  (FARDELLI FITZPATRICK LI)

- EXTEND & COMPLEMENT THIS APPROACH TO $\mathcal{N}=4$ SYM $\rightarrow$ HOW DO TRIPLE TRACES ORGANISE?

  (SEE FOR INSTANCE HOMRICH, SIMMONS DUFFIN, VIEIRA)

- UNDERSTAND SYSTEMATICS OF LARGE N EXPANSION