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UNIVERSE+ Online Seminar

Agnese Bissi
“AdS Amplitudes from CFT”

Part 2

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AdS Loops Amplitudes from CFT

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(ICTP & Uppsala University)

Based on:

Work in progress w/ G. Fardelli & M.R. Khansari

& also different collaborations w/ G. Fardelli, A. Georgoudis, A. Manenti

STRATEGY

- INVERSION FORMULA: IT IS POSSIBLE TO INVERT THE OPE AND FROM THE SINGULARITIES RECONSTRUCT THE OPE DATA.

$$c_{\Delta,\ell} = \int_0^1 dz\, d\bar{z}\;\; \mu(z,\bar{z})\;\; \mathrm{dDisc}\left[\, \mathcal{G}(z,\bar{z}) \right]$$

[S. CARON HUOT]

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$$\mathrm{dDisc}\left[\mathcal{G}(z,\bar{z})\right] = \mathcal{G}^{\mathrm{Eucl}}(z,\bar{z}) - \frac{\mathcal{G}^{\curvearrowleft}(z,\bar{z}) - \mathcal{G}^{\curvearrowright}(z,\bar{z})}{2}$$

ANALYTIC CONT. AROUND $\bar{z}=1$.

[S. CARON HUOT]

$u = z\bar{z}$

$v = (1-z)(1-\bar{z})$

$$c_{\Delta,\ell} \underset{\Delta \to \Delta^*}{\sim} \frac{c^2_{\Delta^*,\ell}}{\Delta - \Delta^*}$$

HAS POLES AT THE DIMENSION OF EXCHANGED OPERATORS AND RESIDUE THE SQUARE OF THE OPE COEFFICIENT.

$$\boxed{\text{SINGULARITIES AROUND } \bar{z}=1 \longrightarrow \text{OPE DATA } (\ell \geq 2)}$$

TWO USEFUL RELATIONS:

$$\sim \left(\frac{v}{u}\right)^{\lambda}$$

$$\mathrm{dDisc}\left[\left(\frac{1-\bar{z}}{\bar{z}}\right)^{\lambda}\right] = \left(\frac{1-\bar{z}}{\bar{z}}\right)^{\lambda} 2\sin^2(\pi\lambda)$$

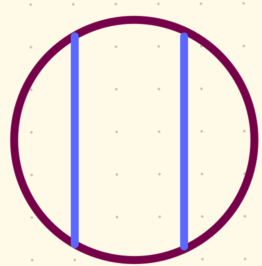
$$\mathrm{dDisc}\left[\log^k(1-\bar{z})\right] = 2\pi^2 k(k-1)\log^{k-2}(1-\bar{z}) + \ldots$$

$$\sim \log^k v$$

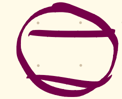
LET US GO BACK TO OUR SETUP

- LEADING ORDER N^0 : GENERALIZED FREE FIELDS

$$\mathcal{G}^{(0)}(u,v) = 1 + u^{\Delta_\phi} + \left(\frac{u}{v}\right)^{\Delta_\phi}$$



- UNDERSTAND HOW TO FIND $a^0_{n,\ell}$ AND Δ WITHOUT KNOWING THE CORRELATOR

- CONSIDER $\theta \times \theta \sim 1 + \ldots$  

- $\mathcal{G}^{(0)}(u,v) \subset a_{0,0}\, u^{0}\, g_{0,0}(u,v) = 1$

- USE CROSSING $\mathcal{G}(u,v) = \left(\frac{u}{v}\right)^{\Delta_\theta} \mathcal{G}(v,u)$

$$1 = \left(\frac{u}{v}\right)^{\Delta_\theta} \mathcal{G}(v,u) \Rightarrow \mathcal{G}(v,u) = \left(\frac{v}{u}\right)^{\Delta_\theta}$$

- THE ONLY TERM WITH $\mathrm{dDisc} \neq 0$ IS

$$\left(\frac{v}{u}\right)^{\Delta_\theta} \rightarrow \begin{array}{l} \Delta = 2\Delta_\theta + 2n + \ell \\ \\ a^{(0)}_{n,\ell} \end{array}$$

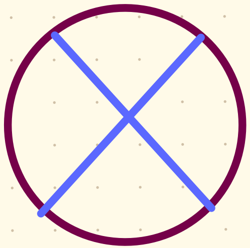
- ORDER N^{-2} : DOES NOT CONTAIN ANY TERM WITH $\text{dDisc} \neq 0$

RESORT TO [HEEMSKERK, PENEDONES, POLCHINSKI, SULLY]

$\downarrow$

$\gamma^{(1)}_{n,0} \neq 0$ $\qquad$ $\gamma^{(1)}_{n,\ell>0} = 0$

$a^{(1)}_{n,0} \neq 0$ $\qquad$ $a^{(1)}_{n,\ell>0} = 0$



- ORDER N^{-2k}

$$\mathcal{G}^{(k)}(u,v) \supset \sum_{n} \frac{1}{2^k k!} u^{\Delta_0+n} a^{(0)}_{n,0} \left(\gamma^{(1)}_{n,0}\right)^k \log^k u \; g_{n,0}(u,v)$$

$$\Big\downarrow \text{ CROSSING}$$

$$\mathrm{dDisc}\left[\log^k v\right] \neq 0$$

COMMENTS:

- AT N^{-4} : THIS IS THE ONLY TERM WITH NON VANISHING dDisc

$\downarrow$

RECONSTRUCT COMPLETELY THE 4 POINT FUNCTION

[AHARONY, ALDAY, AB, PERLMUTTER] [ALDAY, CARON-HUOT]

- DIFFICULTIES IN PERFORMING SUM OVER n AND INTEGRALS.

  ↳ CONTAIN HYPERGEOMETRIC FUNCTIONS & LOG'S.

LIGHT CONE LIMIT : INTEGRAL AROUND $z \to 0 \longrightarrow n=0$

$\bar{z} \to 1 \longrightarrow$ LARGE SPIN

- MANAGE TO FIND THE CONTRIBUTION TO THE ANOMALOUS DIMENSION OF DOUBLE TRACES DUE TO THE HIGHEST $\log u$.

RESULTS

- WE FOUND THE STRUCTURE OF $\gamma_{n,\ell}^{(k)}$ $\qquad \Delta_0 = 2$

$$\gamma_{n,\ell} = \frac{\gamma_{n,0}^{(1)}}{N^2} + \frac{\gamma_{n,\ell}^{(2)}}{N^4} + \dots \qquad \text{EXPANSION IN } N^2$$

$$\gamma_{n,\ell}^{(k)} \underset{\substack{k>1 \\ \ell \to \infty}}{=} \frac{\# + \# \log^{k-2} \ell + \# \log^{k-3} \ell + \dots}{\ell^4} + \dots$$

RECAST THE EXPANSION AS

$$\gamma_{n,\ell} = \frac{\gamma^{(1)}_{n,0}}{N^2} + \frac{K^{(1)}_{n,\ell,N}}{J^4} + \frac{K^{(2)}_{n,\ell,N}}{J^6} + \dots$$

RECAST THE EXPANSION AS

$$\gamma_{n,\ell} = \frac{\gamma^{(1)}_{n,0}}{N^2} + \frac{K^{(1)}_{n,\ell,N}}{J^4} + \frac{K^{(2)}_{n,\ell,N}}{J^6} + \dots$$

$$\underbrace{J^2}_{\text{CONFORMAL SPIN}} = (\ell + \Delta_\theta)(\ell + \Delta_\theta - 1)$$

RECAST THE EXPANSION AS

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$$\Big\downarrow$$

$$K^{(1)}_{0,\ell,N} = -\frac{12}{N^4} J^{-\frac{1}{N^2}} + \text{subleading logs}$$

RECAST THE EXPANSION AS

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$$K^{(2)}_{0,\ell,N} = \frac{1}{N^4}\left(8 J^{-\frac{1}{N^2}} - \frac{256}{5} J^{-\frac{8}{5N^2}}\right) + \ldots$$

$$K^{(1)}_{0,\ell,N} = -\frac{12}{N^4} J^{-\frac{1}{N^2}} + \text{subleading logs}$$

AN EFT PICTURE

INSTEAD OF CONSIDERING LOOPS, WE CAN CONSIDER NEW FIELDS IN AdS WITH LOW TWIST

$$\int_{AdS} \phi\phi\chi \qquad\qquad \chi = \phi^2$$

AT LARGE J, THE ANOMALOUS DIMENSION DUE TO THE SCALAR EXCHANGE IS

$$\gamma \sim -a_{\phi\phi\chi} \frac{2\Gamma(\Delta\phi)^2\,\Gamma(\Delta\chi)}{\Gamma(\Delta\phi - \frac{\Delta\chi}{2})^2\,\Gamma(\Delta\chi/2)^2} \frac{1}{J^{\Delta\chi}}$$

$$\Delta_X = 2\Delta + \varepsilon \gamma_{\phi^2} \qquad (\Delta = 2)$$

$$\gamma \sim \frac{1}{J^4} \sum_{n=0} \varepsilon^{n+2} a^{(0)} \left(\gamma^{(1)}_{\phi^2}\right)^{n+2} \frac{3(-1)^n \log^n J}{\Gamma(n+1)}$$

$\downarrow$

EXACTLY REPRODUCE OUR RESULTS AT LEADING ORDER!

CFT CORRELATORS

$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle =$$

$$\mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \dots$$

$$\downarrow$$

dDisc

$$\downarrow$$

$$\gamma^{(k)}_{n,\ell}\,,\ a^{(k)}_{n,\ell}$$

$$\xrightarrow[\text{LIMIT}]{\text{FLAT SPACE}}$$

AdS AMPLITUDES

$$A_{10}(s,t) =$$

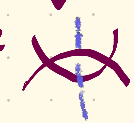
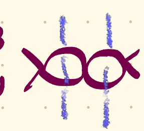
G_N  $+\ G_N^2$  $+\ G_N^3$ 

$+ \dots$

$$\downarrow$$

CUTS & DISCONTINUITIES

$$\downarrow$$

G_N  $+\ G_N^2$  $+\ G_N^3$ 

$+ \dots$

HOW TO MAKE THE CONNECTION:

MELLIN SPACE

$$\mathcal{G}(u,v) = \frac{1}{(4\pi i)^2} \int_{-i\infty}^{+i\infty} ds\, dt\; M(s,t)\; u^{t/2}\, v^{\frac{\hat{u}-2\Delta_\phi}{2}}\, \Gamma^2\!\left(\frac{2\Delta_\phi - t}{2}\right) \Gamma^2\!\left(\frac{2\Delta_\phi - s}{2}\right) \Gamma^2\!\left(\frac{2\Delta_\phi - \hat{u}}{2}\right)$$

$$\hat{u} = 4\Delta_\phi - s - t$$

- $M(s,t)$ CAN BE INTERPRETED AS AdS AMPLITUDE

[PENEDONES]

- AT ORDER N^{-2} $\mathcal{M}(s,t) = \text{const}$

N^{-4} $$\mathcal{M}(s,t) = \sum_{n=0}^{\infty} \frac{R_n^{(1)}(s)}{t-(2\Delta_\phi+2n)} + \text{crossed}$$

- WHAT WE RECONSTRUCT IS AT ORDER N^{-2k}

$$\mathcal{M}^{R}_{k\text{-LOOP}} = \sum_{n=0}^{\infty} \frac{R_n^{(k)}(s)}{(t-(2\Delta_\phi+2n))^k}$$

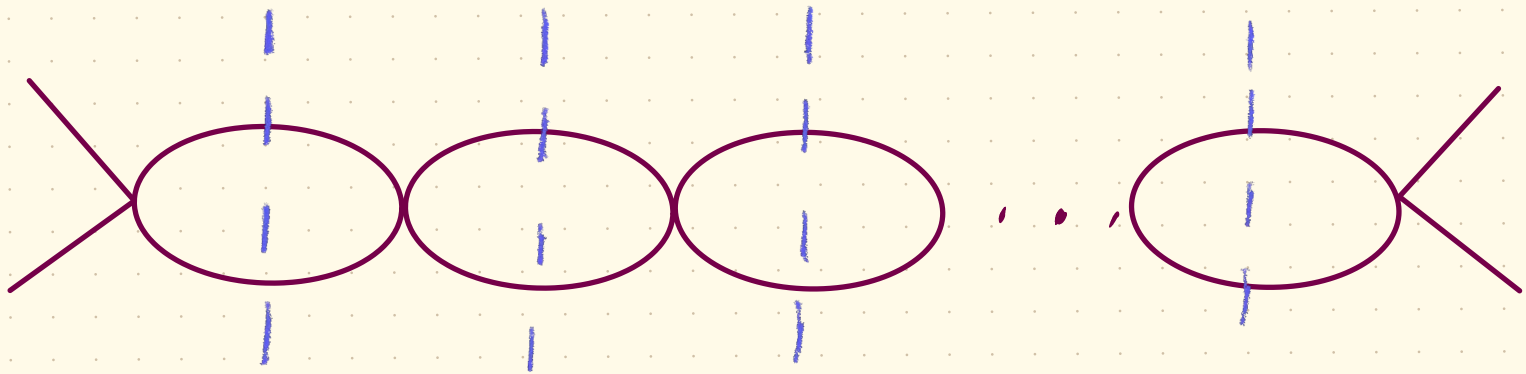
$$\downarrow$$

FIX THE LARGE ℓ EXPANSION

OF $\gamma^{(k)}_{0,\ell>0}$

- FROM THE MELLIN DESCRIPTION, IT IS POSSIBLE TO TAKE THE FLAT SPACE LIMIT AND MATCH IT WITH THE CORRESPONDING FLAT SPACE AMPLITUDES.

- TO WHICH PART OF THE AMPLITUDE DOES THE $\log^k u\, f(u,v)$ CORRESPOND TO?



UNITARITY CUT : CUT ON THE FEYNMAN DIAGRAM
PUTTING THE PROPAGATOR ON SHELL

WE CHECKED IT IN FLAT SPACE AND WE CONJECTURE
THAT IT WORKS IN THE SAME WAY IN AdS

FOR N=4 SYM AB, G.FARDELL,
A.GEORGOUDIS

SUMMARY & FUTURE DIRECTIONS

- HIGHEST $\log \mu$ IN CFTs WITH ϕ^4 TYPE INTERACTION

  $\updownarrow$

  ITERATED CUTS IN THE AMPLITUDE PICTURE

- EMERGENCE OF HIGHER TRACE OPERATORS

- EXTEND THE DISCUSSION TO ϕ^3 TYPE INTERACTIONS (IN PROGRESS)

- UNDERSTAND THE EMERGENCE OF HIGHER TRACE OPERATORS IN SUSY THEORIES.

  - STUDY HIGHER LOOPS OR - EXTERNAL DOUBLE TRACE
    - $\frac{1}{2}$ BPS
    - $\frac{1}{4}$ BPS
  - HIGHER POINT FUNCTIONS