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UNIVERSE+ Online Seminar

Jeffrey Backus

“Half-Ladder partial waves with some applications to string theory”

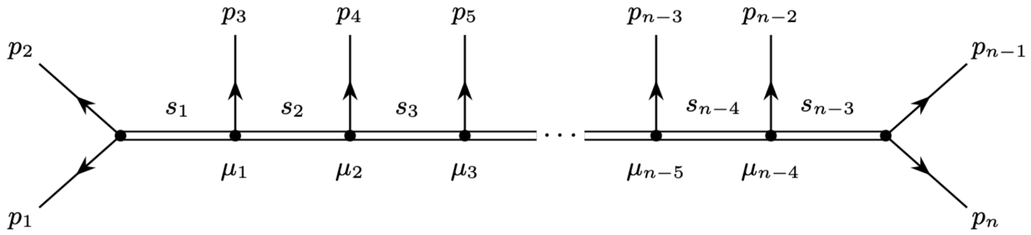
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$$\mathcal{A}_n = \int_0^\infty \prod_e \frac{dy_e}{y_e} \prod_C u_C(y)^{\alpha' X_C}$$

Half-Ladder Partial Waves

With Some Applications to String Theory

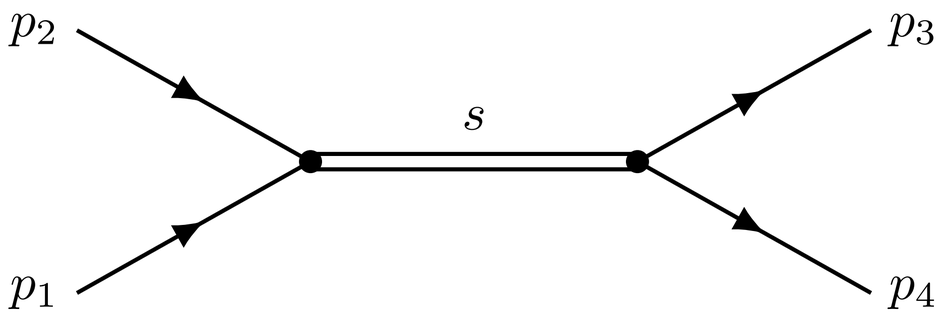
Jeffrey Backus

Princeton University

UNIVERSE+ Seminar 10/14/26

Based on: 2609.28617

Review: at **four-points,** consider the diagram:



For a fixed internal spin-s, the diagram is proportional to a **Gegenbauer**:

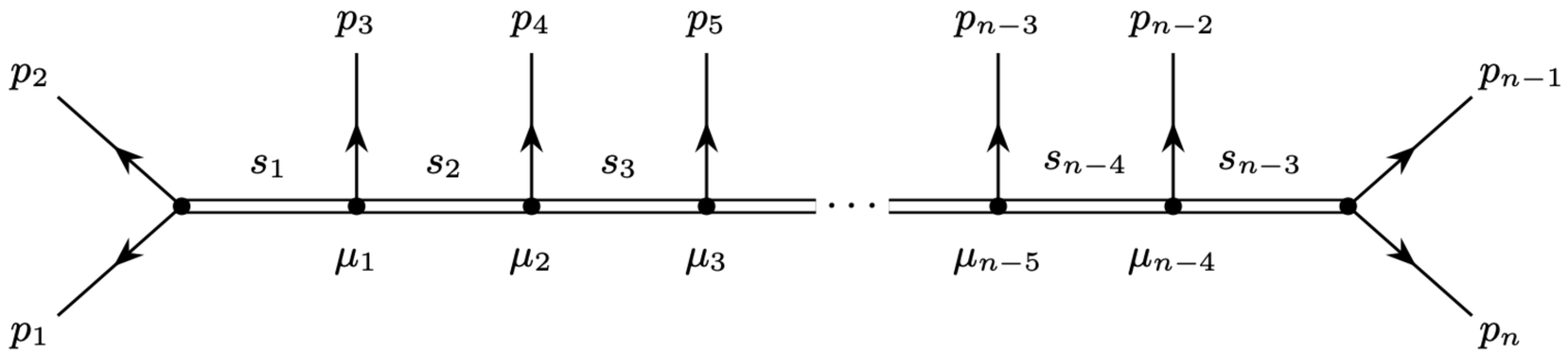
$$C_s^{\left(\frac{d-3}{2}\right)}(\cos\theta)$$

Between incoming and outgoing particles in the center-of-mass frame

These are the **four-point partial wave basis elements**

$$\sum_{s=0}^{\infty} y_s C_s^{\left(\frac{d-3}{2}\right)}(\cos\theta)$$

We want to do the same for the “**half-ladder**” Feynman diagram:



Our partial-wave basis will be labeled by **two sets of quantum numbers**:

s_i = irrep of $SO(d-1)$ belonging to a **particle**

μ_i = irrep of $SO(d-2)$ belonging to a **vertex**

To describe the process, we need to understand the **kinematics:**

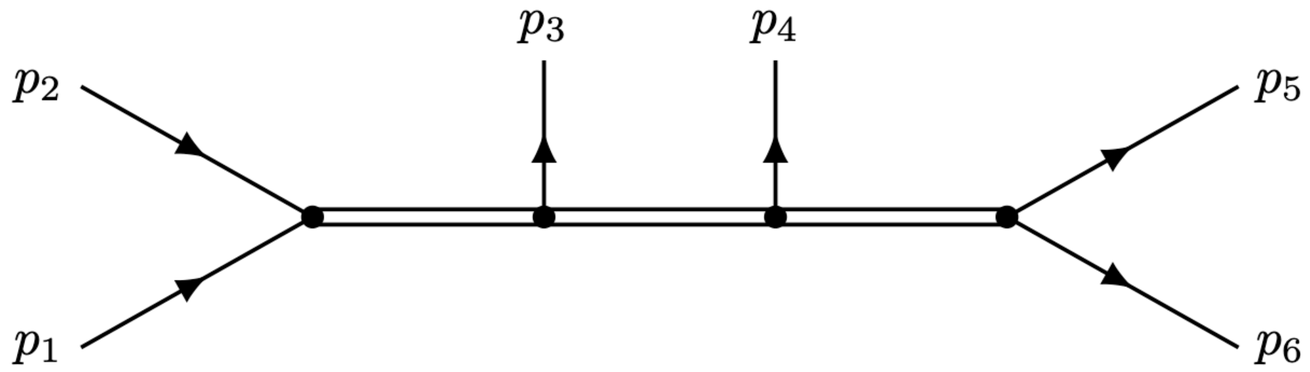
Internal particles fixed to be "on-shell" $X_{1,i} = -M_i^2$

$$\frac{n(n-3)}{2} - (n-3) = \frac{(n-2)(n-3)}{2}$$

Independent kinematic variables $X_{i,j}$ for n-point scalar scattering

Final count: **triangle numbers!**

We will parameterize these by a set of **angles** $\theta_{i,j}$.



$\theta_{2,3}$ $\theta_{3,4}$ $\theta_{4,5}$ **Polar angles**: between $\vec{p}_i$ and $\vec{p}_{i+1}$

$\theta_{2,4}$ $\theta_{3,5}$ **Depth-1 azimuths**: between $(\vec{p}_i, \vec{p}_{i+1})$ and $(\vec{p}_{i+1}, \vec{p}_{i+2})$

$\theta_{2,5}$ **Depth-2 azimuth**: between $(\vec{p}_i, \vec{p}_{i+1}, \vec{p}_{i+2})$ and $(\vec{p}_{i+1}, \vec{p}_{i+2}, \vec{p}_{i+3})$

We enforce that **half-ladder basis element** R_n satisfies

$$-\nabla^2_{S^{d-2}} R_n(\theta) = s_i(s_i + d - 3)R_n(\theta)$$

This identifies each s_i with the quantum mechanical **spin** of the internal particle.

It also gives us an **orthogonal basis** μ_i for the internal vertices.

Three functions result from these equations:

- **Gegenbauers**

$$C_s^{\left(\frac{d-3}{2}\right)}(\cos\theta)$$

- **Associated Gegenbauers:**

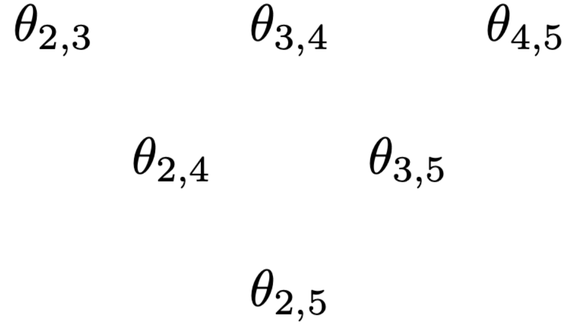
$$A^d_{s,\mu}(\theta) \;\equiv\; (\sin\theta)^\mu \; C_{s-\mu}^{\left(\mu+\frac{d-3}{2}\right)}(\cos\theta), \qquad \mu = 0,1,\ldots,s\,.$$

- **Special T functions:** sums of Gegenbauers

$$T^d_{s,\mu_1,\mu_2,l}(\theta) \;\propto\; (\sin\theta)^{|\mu_1-\mu_2|} \sum_j \gamma_j \, C^{\left(\frac{d-3}{2}+l+|\mu_1-\mu_2|+j\right)}_{s-|\mu_1-\mu_2|-l-2j}(\cos\theta)$$

All have nice orthogonality properties as desired.

Let us put them together for the **six-point** example:



$$R^{(6)}_{s_1 s_2 s_3;\,\mu_1\mu_2} = A^{d}_{s_1,\mu_1}(\theta_{2,3})\; F^{d}_{s_2,\mu_1,\mu_2}(\theta_{3,4},\theta_{2,4},\theta_{3,5},\theta_{2,5})\; A^{d}_{s_3,\mu_2}(\theta_{4,5})$$

$$F^{d}_{s_2,\mu_1,\mu_2} = \sum_{l=0}^{\min(\mu_1,\mu_2)} d_l\; T^{d}_{s_2,\mu_1,\mu_2,l}(\theta_{3,4})\, A^{d-1}_{\mu_1,l}(\theta_{2,4})\, A^{d-1}_{\mu_2,l}(\theta_{3,5})\, C_l^{\left(\frac{d-5}{2}\right)}(\cos\theta_{2,5})$$

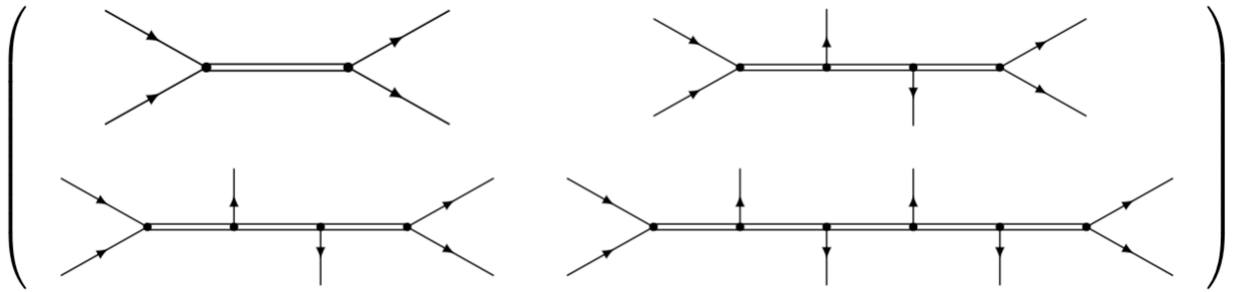
d_l are **known** coefficients given by the Gegenbauer addition theorem.

Application: consistency conditions on **“stringy”** integrals

$$\mathcal{A}_n = \int_0^\infty \prod_e \frac{dy_e}{y_e} \prod_C u_C(y)^{\alpha' X_C}$$

Must obey **positivity** conditions from **unitarity.** But for internal particles (2,1) and (1,0):

$$\det\left(\quad \right) = -\frac{1}{8(d-3)^2} < 0$$



So “Z-theory” **fails to define a consistent quantum mechanical theory in all d!**

Thank you!