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UNIVERSE+ Online Seminar

Andrzej Pokraka
“Hidden zeros of the cosmological wavefunction”

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Hidden Zeros of the Cosmological Wavefunction

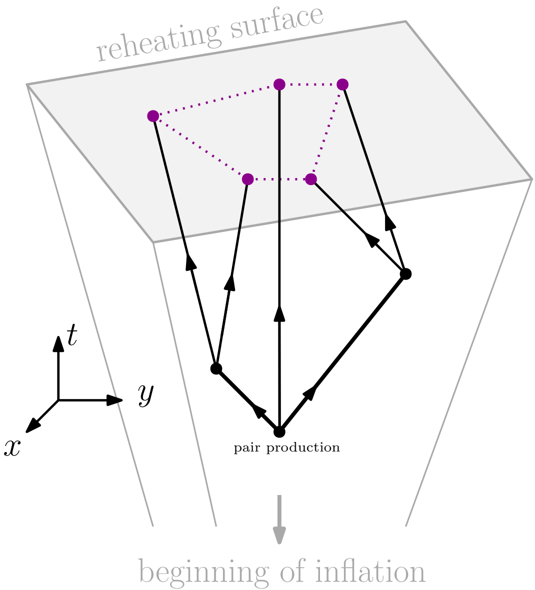
Andrzej Pokraka

Based on 2503.23579

With Shounak De, Shruti Paranjape, Marcus Spradlin, Anastasia Volovich

Universe+ collaboration online seminar series

Cosmological correlators



Cosmological correlators: measure spatial correlations of LSS or temperature in the universe

Consequence of quantum fluctuations imprinted into reheating surface after inflation

$\sim$ initial conditions for time evolution of universe

Wavefunction coefficients

Cosmological correlators are path integrals whose kernel is called the *wavefunctional of the universe* Ψ

$$\langle\varphi(\mathbf{x}_1)\cdots\varphi(\mathbf{x}_N)\rangle = \int \mathcal{D}\varphi\, \varphi(\mathbf{x}_1)\cdots\varphi(\mathbf{x}_m)\, |\Psi[\varphi]|^2$$

Ψ expanded for small fluctuations in the field $\varphi \to \varphi + \delta\varphi$

$$\Psi[\varphi] := \exp\left[i\sum_m \frac{1}{m!}\int\left(\prod_{i=1}^{m}\frac{\mathrm{d}^d\mathbf{k}_i}{(2\pi)^d}\,\varphi_{\vec{k}_i}\right) \times \delta^{(d)}\left(\mathbf{k}_1+\cdots+\mathbf{k}_m\right)\psi_m^{(\ell)}\left(\mathbf{k}_1,\ldots,\mathbf{k}_m\right)\right]$$

The expansion coefficients are called *wavefunction coefficients (WFCs)* — cosmological analogues of *scattering amplitudes*

The toy model

Conformally-coupled scalar field in a power-law FRW cosmology with non-conformal polynomial interactions in $(d+1)$-dimensional spacetime
[N. Arkani-Hamed, J. Maldacena '15, N. Arkani-Hamed, P. Benincasa, A. Postnikov '17, N. Arkani-Hamed, A. Hillman '19, + many more]

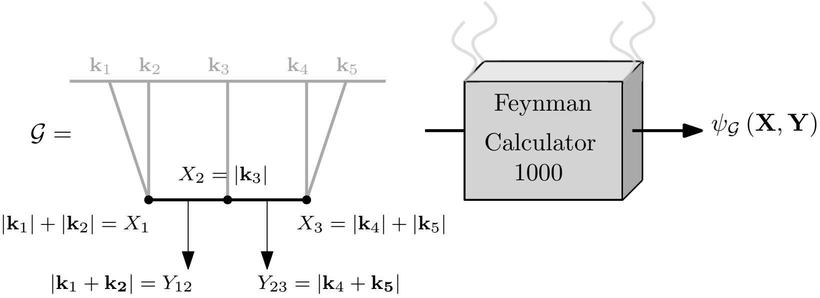
$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + \mathrm{d}x_i\mathrm{d}x^i\right] \qquad a(\eta) = \frac{1}{\eta^{1+\varepsilon}} \begin{cases} \varepsilon = 0 & (\mathrm{dS}) \\ \varepsilon = -1 & (\mathrm{flat}) \\ \varepsilon = -2 & (\mathrm{RD}) \\ \varepsilon = -3 & (\mathrm{MD}) \end{cases}$$

WFC's for any ε: integrate flat-space WFC against a kernel
[N. Arkani-Hamed, D. Baumann, A. Hillman, A. Joyce, H. Lee, G.L. Pimentel '23; S. De, **AP** '23; B. Fan and Z.-Z. Xianyu '24; P. Benincasa, G. Brunello, M.K. Mandal, P. Mastrolia, F. Vazão '24; C. Fevola, G.L. Pimentel, A-L. Sattelberger, T. Westerdijk '24; + many more]

$$\psi_{\varepsilon,\mathcal{G}}(\underbrace{\mathbf{X},\mathbf{Y}}_{\text{kin. data}}) := \int_0^\infty \mathrm{d}x_1\, x_1^\varepsilon \cdots \int_0^\infty \mathrm{d}x_n\, x_n^\varepsilon\, \psi_{\mathcal{G}}(\mathbf{x}+\mathbf{X},\mathbf{Y})$$

Computing flat-space wavefunction coefficients

Wavefunction coefficients can be computed using Feynman diagrams



Feynman diagrams are *not* efficient and do not expose the hidden simplicity of WFCs!

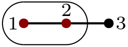
Graph tubings to the rescue

Efficient formula from graph tubings that enhances our mathematical understanding of WFCs [N. Arkani-Hamed, P. Benincasa, A. Postnikov '17]

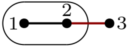
$$\psi_{\mathcal{G}} = \sum_{\mathsf{T}_{\text{max,comp}}} \prod_{\tau \in \mathsf{T}} \frac{1}{S_\tau}$$

A tube, τ, is a set of two sets $\tau = \{ \mathcal{V}_\tau \, , \, \mathcal{E}_\tau \}$

set of vertices of $\mathcal{G}$ enclosed by τ



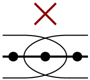
set of edges of $\mathcal{G}$ crossed by τ



Tube variables: $S_\tau = \sum_{v \in \mathcal{V}_\tau} X_v + \sum_{e \in \mathcal{E}_\tau} Y_e$

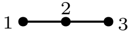
Maximal compatible tubing $\mathsf{T} = \{\tau_1, \dots, \tau_{2n+\ell-1}\}$

- Compatible: $\tau_i \cap \tau_j = \varnothing \; \forall \; \tau_i, \tau_j \in \mathsf{T}$
- Maximal: $|\mathsf{T}| = 2n + \ell - 1$



Example: 3-site chain graph

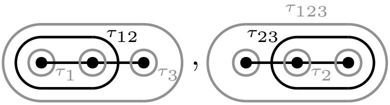
$\mathcal{G} =$



(3-chain graph)

$$\Downarrow$$

$\mathsf{T}_{\mathrm{max,comp}} \in \Big\{$



$\Big\}$

$$\Downarrow$$

$$\psi_{\text{3-chain}} = \frac{1}{S_1 S_2 S_3 S_{123}} \left(\frac{1}{S_{12}} + \frac{1}{S_{23}} \right)$$

Label tubes τ and corresponding S_τ by vertices they encircle

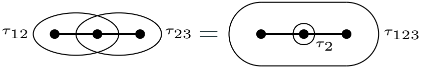
Cosmological hyperplane arrangements are degenerate

Realization of S_τ in terms of X_v and Y_e not important for this talk

Important: tube varibles S_τ are constrained; tubes tell us how

$$\boxed{S_{\tau_i} + S_{\tau_j} = S_{\tau_i \cup \tau_j} + \sum_{\tau \in \tau_i \cap \tau_j} S_\tau} \qquad (\star)$$

3-chain example:



$$\Longrightarrow \quad S_{12} + S_{23} = S_{123} + S_2$$

Multiple representations for the vanishing loci of $\psi_{\text{3-chain}}$

$$\psi_{\text{3-chain}} = \frac{1}{S_1 S_2 S_3 S_{123}} \frac{S_{12} + S_{23}}{S_{12} S_{23}} = \frac{1}{S_1 S_2 S_3 S_{123}} \frac{S_{123} + S_2}{S_{12} S_{23}}$$

The zeros and near zero splitting of amplitudes

Recent advances in our understanding of the zeros of amplitudes and how they split or factor near these zeros
[S. Telen '25; B. Giménez Umbert, B. Sturmfels, '25; N. Arkani-Hamed, Q. Cao, J. Dong, C. Figueiredo, S. He, '24; F. Cachazo, N. Early, B. Giménez Umbert '22; $\cdots$ L.J. Dixon, Z. Kunszt, A. Signer '99; A. D'Adda, S. Sciuto, R. D'Auria, F. Gliozzi '71; L. Adler '65 $\cdots$]

While factorization near poles well follows from unitarity, the physical origin of splitting near zeroes remains mysterious

Ultimately want to understand the zeros of $\psi_{\mathcal{G}}$ and its splittings

This talk: satisfied by understanding the zeros of *stripped WFCs* $\tilde{\psi}_{\mathcal{G}}$—simpler piece of ψ_G

Outline

1) Stripped WFCs and graph associahedra
    1a) Combinatorics of stripped tubes organize into graph associahedra
    1b) Construct hyperplane realization of the graph associahedra and relate their canonical functions to stripped WFCs
2) Zeros of the stripped WFCs
    2a) Parametric/flattening zeros
    2b) The adjoint polynomial and wavefunction zeros
    2c) Factorization zeros
3) Connection to the zeros of Tr$[\phi^3]$ amplitudes
4) Conclusion

Stripped WFCs and the graph associahedron

Stripped WFCs

Recall ∃ set of tubes compatible with all other tubes:

$$\mathsf{T}_{\text{triv,comp}} = \{\tau_1, \tau_2, \ldots, \tau_n, \tau_{\text{total}}\}$$

Stripped WFC:

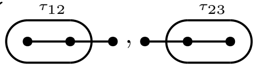
$$\tilde{\psi}_{\mathcal{G}} = \left(\prod_{\tau \in \mathsf{T}_{\text{triv,comp}}} S_\tau \right) \psi_{\mathcal{G}} = \sum_{\tilde{\mathsf{T}}_{\text{max,comp}}} \prod_{\tau \in \tilde{\mathsf{T}}} \frac{1}{S_\tau}$$

$$\tilde{\mathsf{T}} \cap \mathsf{T}_{\text{triv,comp}} = \varnothing$$

$$|\tilde{\mathsf{T}}_{\text{max,comp}}| = n + \ell - 2$$

3-chain example:

$$\tilde{\mathsf{T}}_{\text{max,comp}} \in \{ \quad , \quad \}$$

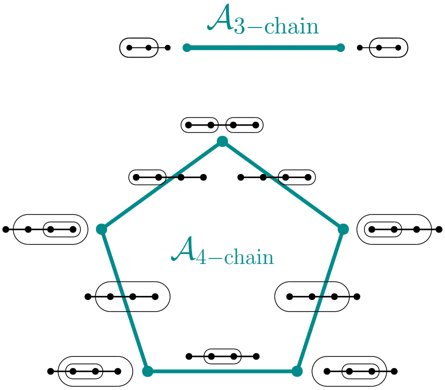


$$\implies \tilde{\psi}_{\text{3-chain}} = \frac{1}{S_{12}} + \frac{1}{S_{23}} = \frac{S_{12} + S_{23}}{S_{12} S_{23}} = \frac{S_{123} + S_2}{S_{12} S_{23}} \qquad (\star)$$

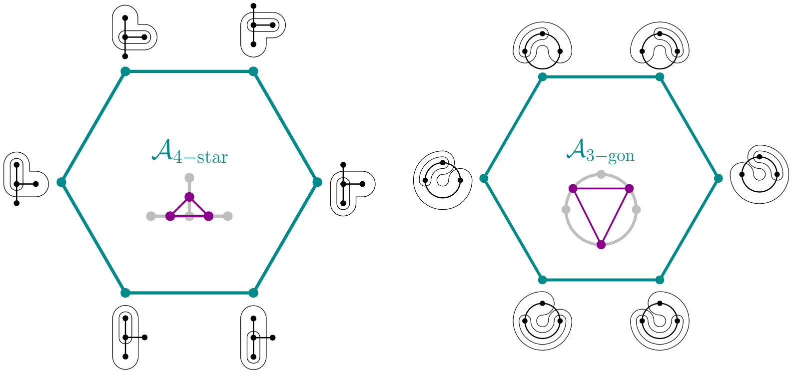
Graph associahedra ($\mathcal{A}_{\mathcal{G}}$) and compatibility of tubings

Stripped maximal compatible tubings correspond to vertices of graph associahedra $\mathcal{A}_{\mathcal{G}}$ [N. Arkani-Hamed, C. Figueiredo, Francisco Vazão '24]

Faces correspond to tubes, codimension-k facets encode compatibility of stripped $(k-1)$-Tubings



Graph associahedra from compatibility of tubings



$\tilde{\psi}_{\mathcal{G}}$ should be the cannonical function of $\mathcal{A}_G$

$\implies$ want hyperplane realization to make this explicit

A graph associahedron for stripped WFCs

Modify the Carr-Devadoss algorithm so that 2-tube S's are our coordinates instead of 1-tube S's

[M. Carr and S. L. Devadoss '05; S. L. Devadoss '06]

Coordinates: $\{S_\tau\}$ where τ is a 2-tube are coordinates of $\mathbb{R}^{m=n+\ell-1}$

Parameters: $\{S_{\text{total}}\} \cup \{S_v\}_{v \text{ an internal site}}$ control specific geometry of $\mathcal{A}_{\mathcal{G}}$

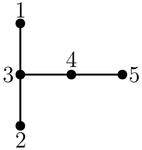
A graph associahedron for stripped WFCs: step 1

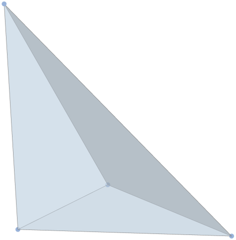
$(m-1)$-simplex by intersecting $\mathbb{R}^m_+ = \{S_\tau \geq 0 : \tau \in \mathcal{T}_2\}$ with hyperplane

$$\sum_{\tau \text{ a 2-tube}} S_\tau = S_{\text{total}} + \sum_{\text{interior sites } v} \# \, S_v$$

(obtain by reducing S_{total} to 1- and 2-tube S's using $(\star)$)

Example (5-star graph):





Coordinates: $(S_{13}, S_{23}, S_{34}, S_{45}) \in \mathbb{R}^4_+$

Parameters: $(S_{12345}, S_3, S_4) \in \mathbb{R}^3_+$

$$S_{13} \geq 0, S_{23} \geq 0, S_{34} \geq 0, S_{45} \geq 0,$$

$$S_{13} + S_{23} + S_{34} + S_{45} = S_{12345} + 2S_3 + S_4$$

A graph associahedron for stripped WFCs: step 2

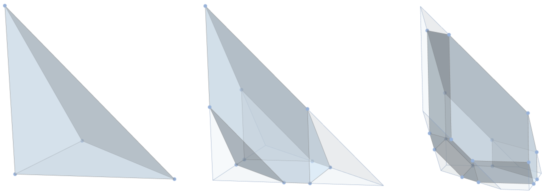
Each $(m-k-1)$-dimensional facet of the simplex is truncated by the hyperplanes $S_{\tau \text{ a } k\text{-tube}} = 0$

$$S_{\tau \text{ a } k\text{-tube}} = \sum_{\substack{\tau':\ 2\text{-tubes} \\ \text{in } k\text{-tube } \tau}} S_{\tau'} - \sum_{\text{interior sites } v} \# S_v + \delta_\tau \geq 0$$

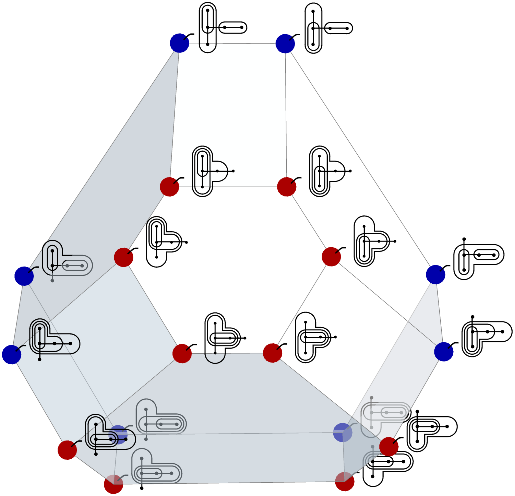
Ensure that polytope is simplicial with correct combinatorial interpretation:

$$S_{\text{total}} \geq S_v \geq \delta_{\tau \in \mathcal{T}_3} \geq 0$$

$$\delta_{\tau_1} + \delta_{\tau_2} \geq \delta_{\tau_1 \cup \tau_2} + \delta_{\tau_1 \cap \tau_2} \text{ when } \tau_1 \cap \tau_2 \neq \emptyset$$

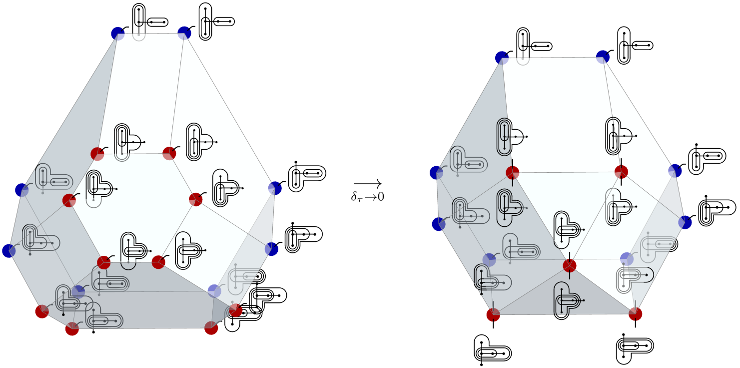
Example (5-star graph): 

Tube compatibility



Linear constraints $(\star)$ *not* satisfied if $\delta_\tau \neq 0$!

The cosmological limit: $\delta_\tau \to 0$

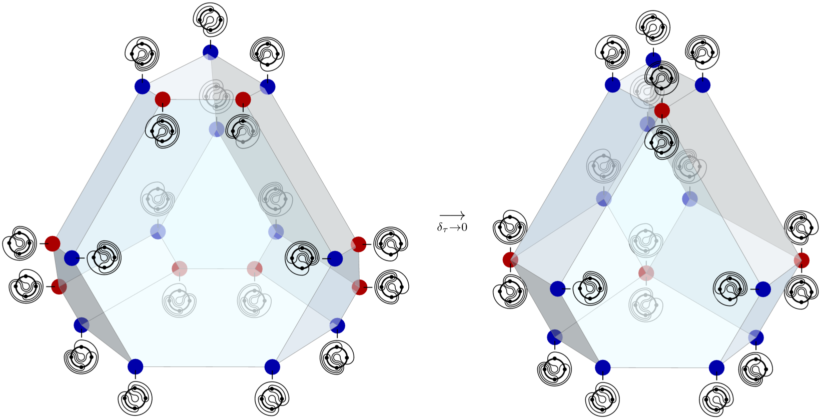


$$\implies \tilde{\psi}_{\mathcal{G}} = \lim_{\delta_\tau \to 0} \hat{\Omega}[\mathcal{A}_{\mathcal{G}}]$$

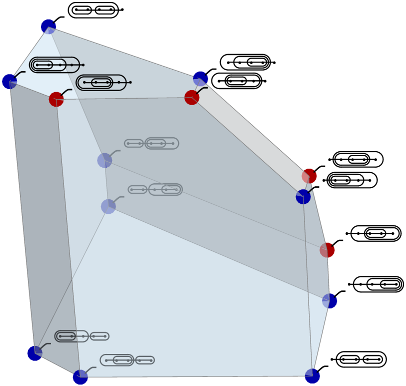
(automatic in the X_v and Y_e parameterization of the S_τ)

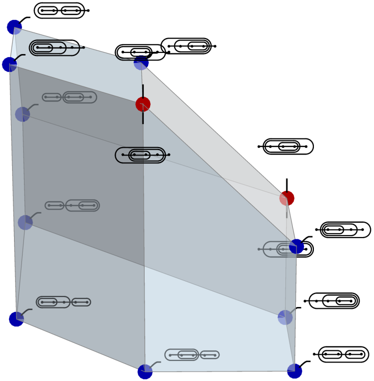
3-dimensional examples

$\lim_{\delta_\tau \to 0} \mathcal{A}_{\mathcal{G}}$ is degenerate when $n + \ell > 4$



3-dimensional examples





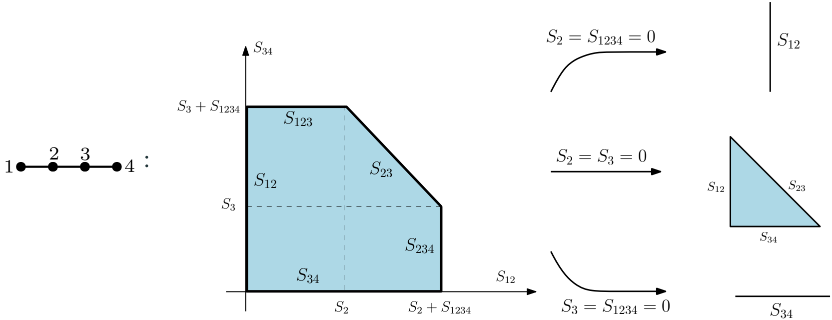
Parametric/flattening zeros

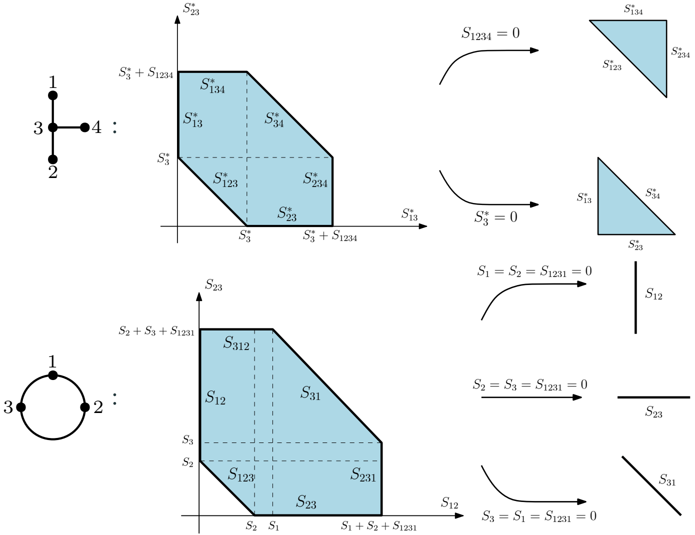
Parametric/flattening zeros

From now on: $\delta_\tau = 0$

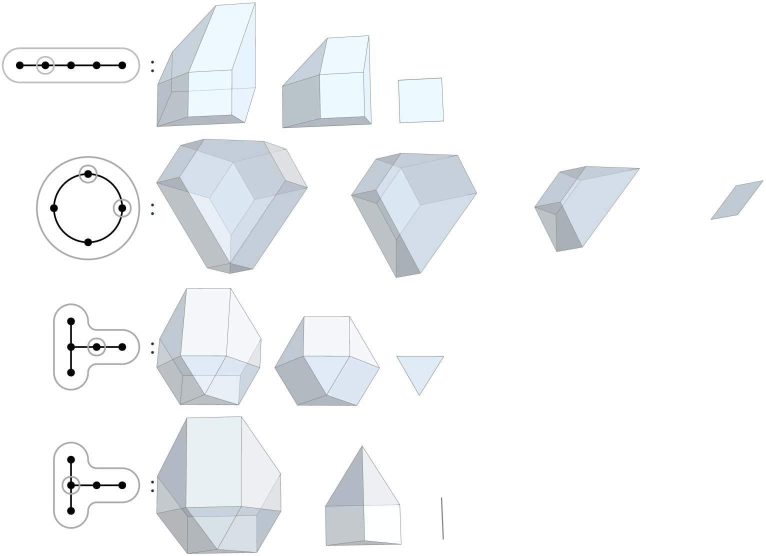
Zeros of Tr$[\phi^3]$ controlled by flattening of the ABHY associahedron

[N. Arkani-Hamed, Q. Cao, J. Dong, C. Figueiredo, S. He '24; N. Arkani-Hamed, Y. Bai, S. He, G. Yan '17]

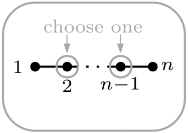



1
3
4
2
:
S^*_{23}
$S^*_3 + S_{1234}$
S^*_{134}
S^*_{13}
S^*_{34}
S^*_3
S^*_{123}
S^*_{234}
S^*_{23}
S^*_{13}
S^*_3
$S^*_3 + S_{1234}$
$S_{1234} = 0$
S^*_{134}
S^*_{234}
S^*_{123}
$S^*_3 = 0$
S^*_{13}
S^*_{34}
S^*_{23}
1
3
2
:
S_{23}
$S_2 + S_3 + S_{1231}$
S_{312}
S_{12}
S_{31}
S_3
S_2
S_{123}
S_{231}
S_{23}
S_{12}
S_2
S_1
$S_1 + S_2 + S_{1231}$
$S_1 = S_2 = S_{1231} = 0$
S_{12}
$S_2 = S_3 = S_{1231} = 0$
S_{23}
$S_3 = S_1 = S_{1231} = 0$
S_{31}

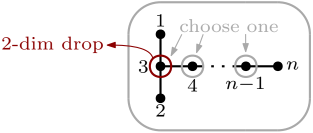
3-dimensional examples



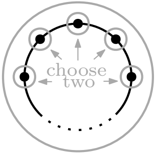
Parametric zero summary



$$: \quad \tilde{\psi}_{n\text{-chain}}\big|_{S_{12\ldots n}=S_v=0} = 0$$

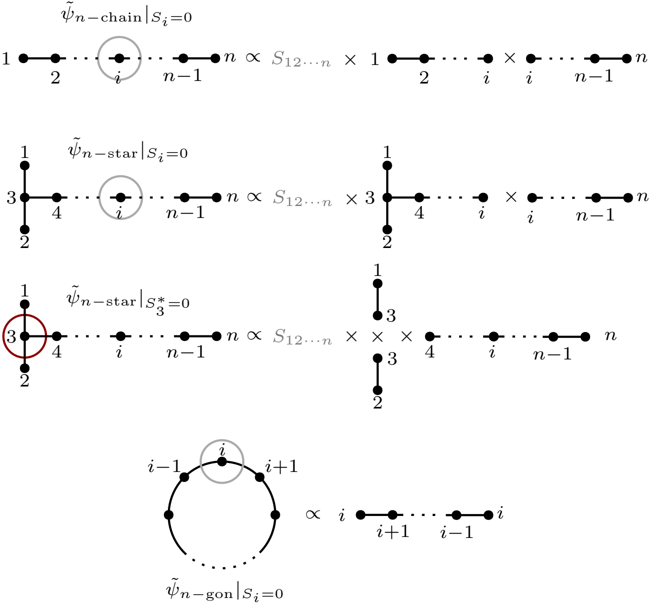


$$: \quad \tilde{\psi}_{n\text{-star}}\big|_{S_{12\ldots n}=S_v=0} = 0$$



$$: \quad \tilde{\psi}_{n\text{-gon}}\big|_{S_{12\ldots n}=S_{v_1}=S_{v_2}=0} = 0$$

Splitting near parametric zeros: Relaxing $S_{\text{tot}} = 0$



The adjoint polynomial and wavefunction zeros

The adjoint polynomial

Adjoint polynomial: polynomial in the numerator of a canonical function after putting everything over a common denominator

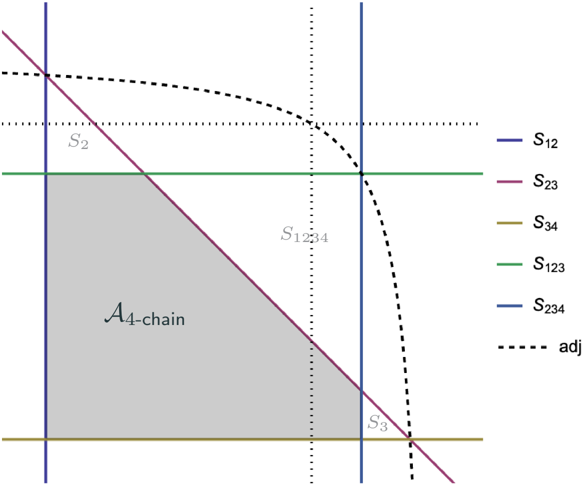
$$\tilde{\psi}_{4-\text{chain}} = \frac{\text{adj}}{S_{12}S_{23}S_{34}S_{123}S_{234}}$$

$$\text{adj} = S_{12}\left(S_{123}\left(S_{23}+S_{34}\right)+S_{234}S_{34}\right)+S_{23}S_{234}\left(S_{123}+S_{34}\right)$$

The zero locus of the adjoint polynomial passes through the points corresponding to the maximal intersection of non-compatible S_τ

Can we use the adjoint polynomial to find linear conditions on the parameters $(S_{1234}, S_2, S_3) \in \mathbb{R}^3_+$ *and at least one* of the coordinates $(S_{12}, S_{23}, S_{34}) \in \mathbb{R}^3_+$?

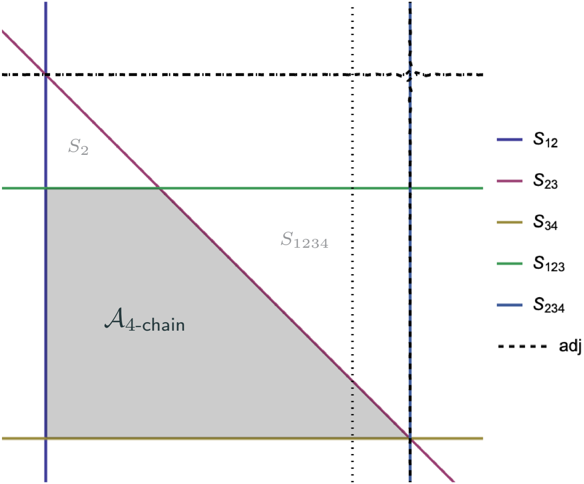
The 4-chain hyperplane arrangement



Degenerating the 4-chain arrangement

$$S_3 = S_{23} + S_{34} - S_{234} = 0$$

$$\text{adj}|_{S_3=0} = S_{234} S_{1234} (S_{1234} + S_2 - S_{34})$$



Degenerating the 4-chain arrangement

Shrinking bottom triangle:

$$S_3 = S_{23} + S_{34} - S_{234} = 0$$

$$\mathsf{adj}|_{S_3=0} = S_{234} S_{1234} (S_{1234} + S_2 - S_{34})$$

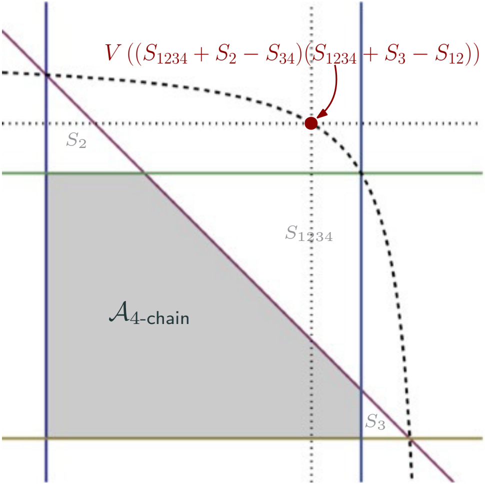
Shrinking upper triangle:

$$S_2 = S_{12} + S_{23} - S_{123} = 0$$

$$\mathsf{adj}|_{S_2=0} = S_{123} S_{1234} (S_{1234} + S_3 - S_{12})$$

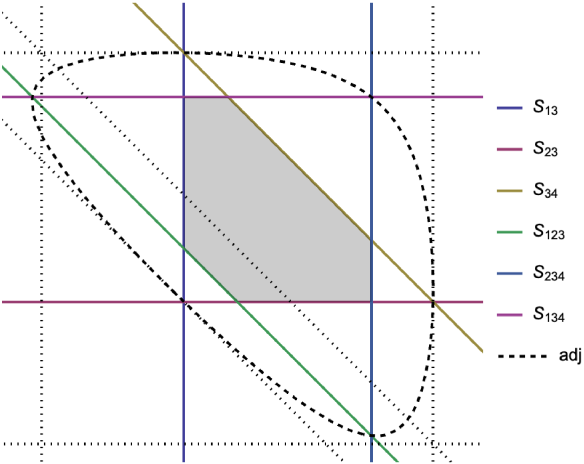
Shrinking middle triangle: rediscover only parametric zeros

The 4-chain wavefuction zero



Zero for both $\tilde{\psi}_{\mathcal{G}}$ and $\psi_{\mathcal{G}}$!

The wavefuction zero is only for n-chains



The wavefuction zero and splitting for n-chains

The wavefunction zero (only exists for chains):

$$\tilde{\psi}_{n-\text{chain}}\big|_{\mathcal{S}_1=\mathcal{S}_2=\cdots=\mathcal{S}_{n-2}=0} = 0$$
$$\mathcal{S}_1 := S_{12\cdots n-1} + S_{23\cdots n}$$
$$\mathcal{S}_{i=2,\ldots,n-2} := S_{12\cdots n-1} - S_{1\cdots i} + S_{i+1\cdots n}$$

Zero for both $\tilde{\psi}$ and ψ!

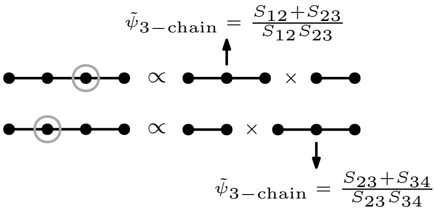
Splitting:

$$\text{relax } \mathcal{S}_{i=2,3,\ldots,n-2} = 0 : \propto \mathcal{S}_i \times \tilde{\psi}_{i\text{-chain}} \times \tilde{\psi}_{(n-i)\text{-chain}}$$
$$\text{relax } \mathcal{S}_1 = 0 : \propto \mathcal{S}_1 \times \underbrace{\tilde{\psi}_{1\text{-chain}}}_{1} \times \tilde{\psi}_{(n-1)\text{-chain}}$$

Factorization zeros

Factorization zeros: 4-chain again

Combine the splitting near $S_i = 0$ with the wavefunction zero



Shrinking bottom triangle:

$$S_3 = S_{23} + S_{34} - S_{234} = 0$$

$$\text{adj}|_{S_3=0} = S_{234} S_{1234} (S_{1234} + S_2 - S_{34})$$

On the loci $S_3 = 0$, after using $(\star)$

$$S_{1234} + S_2 - S_{34}|_{S_3=0} = S_{12} + S_{23}$$

Factorization zeros: Arbitrary graphs

Set enough internal site tube varibles $S_v = 0$ to factor the graph into a product of chain graphs

Choose a chain graph factor and localize to its wavefunction zero to produce a factorization zero

Connection to Tr$[\phi^3]$ amplitudes

Connection to Tr$[\phi^3]$ amplitudes

From WFCs to amplitudes:

- $S_{i\dots j-1} \mapsto X_{i,j}$ for $|i-j| \geq 2$ (coordinates $\mapsto$ planar variables)
- $S_i \mapsto -p_{i-1} \cdot p_{i+1}$ for $2 \leq i \leq n-1$ (parameters $\mapsto$ non-planar variables)
- $S_{12\dots n} \mapsto -p_1 \cdot p_n$ (parameters $\mapsto$ non-planar variables)
- $(n-1)(n-4)/2$ non-planar variables vanish on support of $(\star)$

where $X_{i,j} = (p_i + \cdots + p_{j-1})^2$ and $X_{i,i+1} = 0$

Under this identification, $\tilde{\psi}_{n\text{-chain}} \leftrightarrow A_{n+1}^{\text{Tr}(\phi^3)}$

$$\tilde{\psi}_{4\text{-chain}} = \frac{1}{S_{12}S_{123}} + \frac{1}{S_{23}S_{234}} + \frac{1}{S_{12}S_{34}} + \frac{1}{S_{123}S_{23}} + \frac{1}{S_{234}S_{34}}$$

$$A_5^{\text{Tr}(\phi^3)} = \frac{1}{X_{1,3}X_{1,4}} + \frac{1}{X_{2,4}X_{2,5}} + \frac{1}{X_{1,3}X_{3,5}} + \frac{1}{X_{1,4}X_{2,4}} + \frac{1}{X_{2,5}X_{3,5}}$$

Zeros of $\tilde{\psi}_{n\text{-chain}}$ are zeros of $A_{n+1}^{\text{Tr}(\phi^3)}$ and vice versa

Discussion and future directions

Recap

Constructed an explicit hyperplane realization of the graph associahedron corresponding to the stripped WFC

For $n + \ell > 4$, polytope is degenerate/non-simplicial in the cosmological limit ($\delta_\tau \to 0$)

Classified the linear zero conditions for the flat-space (stripped) WFCs associated to a single Feynman graph $\mathcal{G}$ into parametric, wavefunction and factorization zeros

For each zero type, we derived how the stripped WFC splits or factors

The wavefunction zeros only exist for the family of n-chain graphs; parametric and factorization zeros are universal

Interesting connection between $\tilde{\psi}_{n\text{-chain}}$ and $A_{n+1}^{\mathrm{Tr}(\phi^3)}$ amplitudes (not covered – see paper)

Discussion and future directions

What are the zeros of cosmological WFCs of a Tr(ϕ^3) model? Are they all produced by flattening the cosmohedron?

Can we construct stringy integrals for WFCs? Cosmological δ-shifts?

What is the physics interpretation of the wavefunction zero? Seem to be multi-soft limits:

$$X_i = |\vec{k}_i| = 0 \text{ for all } 3 \leq i \leq n-2$$
$$Y_{12} = -X_1 - 2X_2\,,\; Y_{n-1,n} = -2X_{n-1} - X_n$$

Why should we expect this behaviour from the physics perspective?

Can WFCs be uniquely fixed from its zeros?